Fibonacci perennis
- From W.A. Mozart to L. Bernstein -

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Nothing could be more exciting, in terms of theoretical exercise, than the attempt to comprehend why certain personalities in various intellectual domains feel the need to assume their art in order to use it ultimately to assert some ideas which belong to their areas of interest. For instance, in the preface to *Iocari serio – Ştiinţă şi artă în gândirea Renaşterii* (*Iocari serio – Science and Art in Renaissance Thought*), Ion Petru Culianu wrote: "The religion historian is not merely an interpreter, but rather a *meta-interpreter*, since the language which represents his domain of operation is interpretation itself. The abstraction level of the meta-language will be proportional to the number of interpretation plays: Heraclitus probably interprets an Orphic myth, Nietzsche interprets Heraclitus and the Orphic myth, Nicolaus Cusanus interprets the play as a world symbol relating to the Orphic myth, Nietzsche interprets Renaissance as an age which interpreted play, and so on.

The conclusion of such an approach is not necessarily agnostic: only in play can the interpretation play be interpreted. Hermeneutics is an *Art*"[1].

It is from a completely opposed direction that Bertrand Russell invokes art, as the mathematician and philosopher who did not in the least share the idea that most people admit and feel the need to have faith in a superior power. But even he, who believed that the human mind would answer, or not, by itself all the dilemmas of the universe, even he felt the need of an alliance with the artistic beauty in order to prove the infallibility of mathematical thought. "Mathematics – he wrote – is not only the ultimate truth, but also the ultimate beauty – a cold, austere beauty, like that of a sculpture, without exerting any attraction on whatever part of our weak character, having sublime purity and capable of such stern perfection which one may only see in the highest art"[2].

We have resorted to these two quotes, where art is involved only as a comparison with the significance of absolute perfection in order to be assimilated to other domains, in this case hermeneutics and mathematics, to the purpose of showing that, no matter how different these two may be, both have deep connections with art, in general, and with the art of sounds, in particular.

Living in the middle of the musical phenomenon (as a professional pianist), we have gladly found, after having read the aforementioned books, that

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[1] Ion Petru Culianu, IOCARI SERIO – ŞTIINŢĂ ŞI ARTĂ ÎN GÂNDIREA RENAŞTERII (IOCARI SERIO – SCIENCE AND ART IN RENAISSANCE THOUGHT *)* (Iaşi: Polirom, 2003), p. 18.

[2] Paul Johnson, INTELECTUALII (THE INTELLECTUALS) (Bucharest: Humanitas, 2006), pp. 281-282.

the association between art (in this case, music) and hermeneutics and mathematics is not only possible, but it offers the possibility of creating a commutative group where music is the neutral element since, as a term of comparison, it may lie on the right, as seen in the two quotations, but also on the left, that is: music, generating interpretations and meta-interpretations, or music, where we can discover the precision and rigour of mathematics.

These are the thoughts that we shall display here, without ever claiming to hold absolute truths, since our profession, as previously declared, is other than art aesthetician or theorist. We have been prompted in this enterprise by two arias on the occasion of a singing recital where we performed as an accompanist: the concert aria “Mia speranza adorata … Ah, non sai qual pena” by W.A. Mozart and Cunégonde's aria “Glitter and be gay”, from the operetta *Candide* by L. Bernstein. Having analysed them (which is not an openly-stated obligation, but rather an implicit one), we have found that in both their structures there lies an undeniable presence of mathematical thought, which brings these arias closer to each another although they were created over two centuries apart. Hence, the title of the article. Without any pretentious intentions, we believe that our decision to express several ideas on the art we are practising is also warranted by the fact that, in the equation of the musical phenomenon, the performer may well be considered an “interface” between the composer and the receiver, and his “performance/interpretation” – given the theoretical arguments it is based upon, and having priority over the receiver's “meta-interpretation” – is worth our consideration, since it is on its framework that is built the first concrete image of the musical work.

From a philosophical and aesthetic point of view, this issue is vast and surpasses our possibilities for an exhaustive presentation. Nevertheless, there is a need for a brief explanation regarding the accents on highlighting the formal structures expressed by numbers which we have chosen, as this is derived not from openly adopting one or another from the many aesthetics, but, since we work in education, we are aware that we cannot pass on knowledge otherwise than in a coherent form, and this is only possible when everything is clear to us in the first place.

From the perspective of language, in his monumental work *Poetica matematică* (*The Mathematical Poetics*), Solomon Marcus proposes, starting from Pius Servien's *Aesthetics*, an analysis of the artistic language by means of the mathematical one. “By approaching the issue in this manner – he says in another book – we may avoid on of the main downsides of traditional research, where the meta-language employed is of the same type as the investigated language. Therefore, the language employed should necessarily be of a different type from the artistic one”[3].

[3] Solomon Marcus, ARTĂ ŞI ŞTIINŢĂ (ART AND SCIENCE) (Bucharest: Eminescu, 1986), p. 91.

In analysing the arias previously mentioned we shall try such an approach, but not before stating our standpoint in the controversial issue of style, when we refer to composers who wrote music in tonal language. The main parameters of this language in which they expresses themselves (innovated), each in his own way (style), the vocabulary, morphology, syntax or form could be rendered mathematically as follows:

- 12 sounds set in the temperate chromatic scale: a geometric progression with the ratio $\sqrt[12]{2} = 1.05946$. Of these, 7 were initially used (set in the diatonic tonal scale) and, gradually, a total of 12 was reached (the chromatic scale);
- the durations (between 6 and 8 values), set in a geometric progression with ratio 2;
- the meter (standard), in reference to which the successions of rhythmic values are set; it remains constant or may be altered;
- the chords, set by thirds, are hierarchized in the tonal scale;
- a set of rules to settle any dissonances into consonances;
- a set of rules regarding the succession order of the parts of a form, as well as the outlines of the forms: the lied, the menuet, the rondo, the sonata and so on (all this is based upon the measuring of the parts and establishing the relationships between the parts, between the parts and the whole).

Many of these will be associated to two mathematical notions: the string and the algebraic group structure which we present next without further demonstrations. Actually, these are taught in high school, and the division in extreme and mean ratio is taught in secondary school.

1. The Fibonacci string

$a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2}, n \geq 3$, where $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1+\sqrt{5}}{2} = \Phi = 1{,}618\ldots$(the golden number)

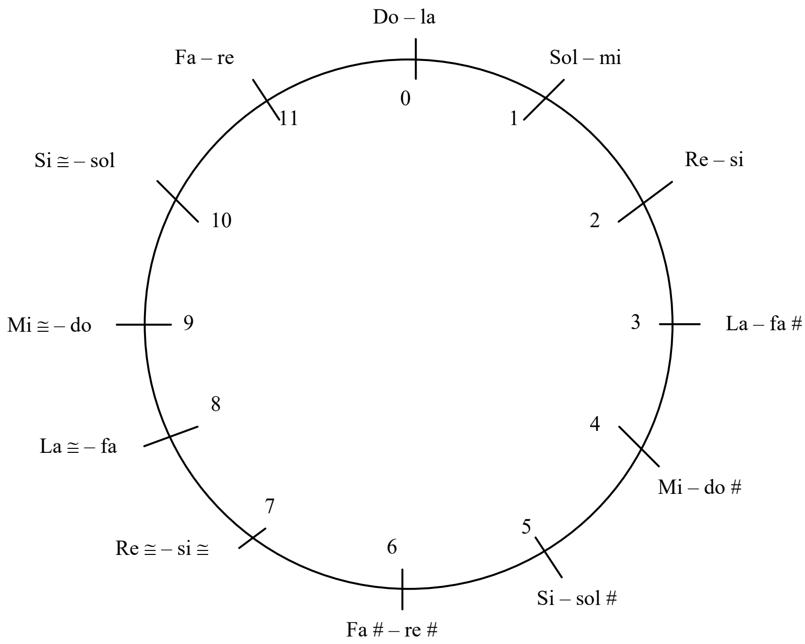
2. Group – a set of elements among which an operation was defined which makes that one element of the set corresponds to two of its elements so that three characteristics are fulfilled:

a) associativity, that is, A*(B*C)=(A*B)*C;

b) have a neutral element so that A*E=E*A=A;

c) each element has a corresponding reversed element, that is, $A*A^{-1}=A^{-1}*A=E$. If it is also commutative, A*B=B*A, it is called abelian group.

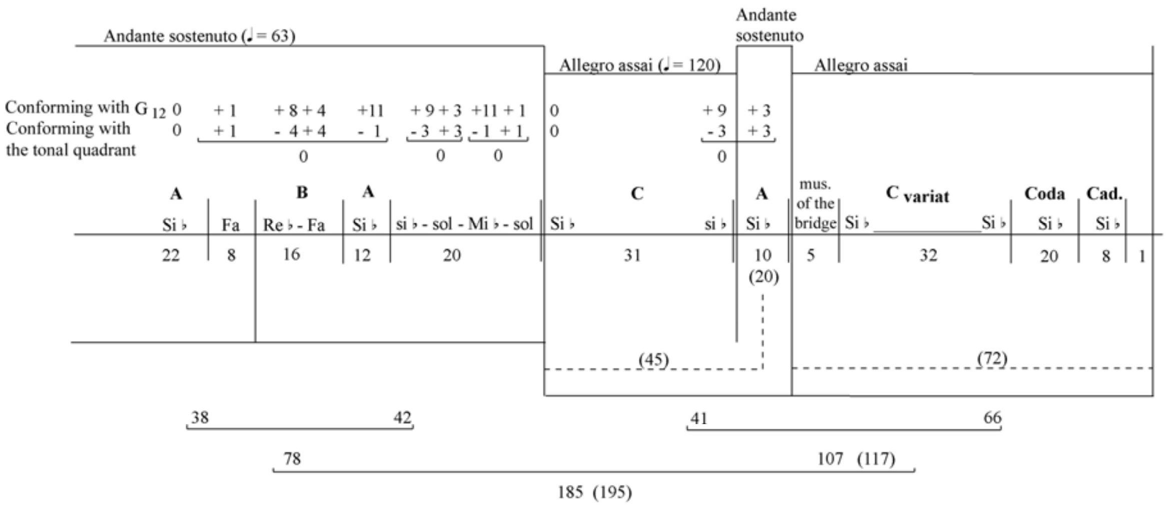
We will associate the group of the congruence classes modulo 12, defined on addition, to the analysis of the tonal level of the analysed works, and we will set the tonalities in the order of the fifths ascending on the tonal quadrant.

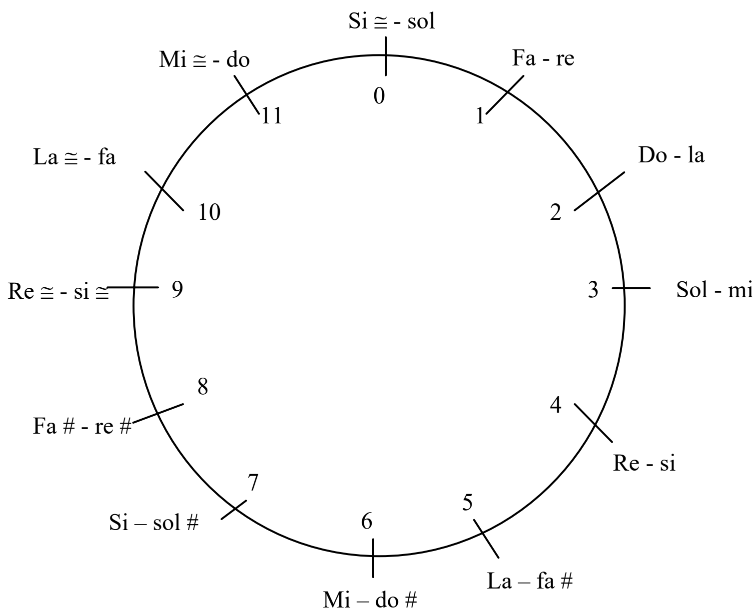


+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10

If Bach and Haydn imposed the tonal language as a form of artistic communication, the oeuvre of W.A. Mozart (1756 – 1791) meant its stability and establishment. It is said that, most of time, the composer first drew up an outline (the tonal plan) and only after that he would write the composition, neatly and correction-free. This made us very curious as to how the outline of the concert aria "Mia speranza adorata..." might have looked like, starting from the pillars of harmony which later gave birth to the melodic motion and the architecture of the form, which, eventually, make up the defining elements of style.

RONDO





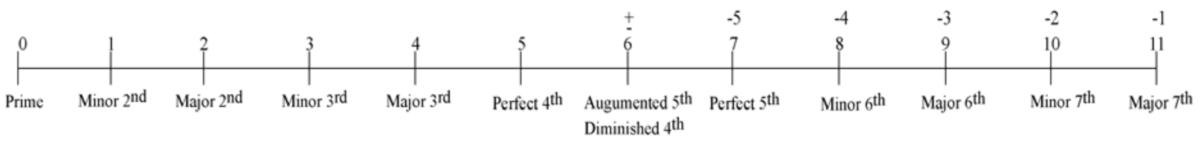
Attempting to rebuild Mozart's path, we drew the outline of the form according to his very own indication, RONDO, and we concluded that it didn't really fit. First of all, there are two movements, Andante sostenuto (♩ = 63) and Allegro assai (♩ = 120) which is interrupted, at one point, by the brief re-occurrence of the first idea in Andante, and then, until the end, Allegro. The letters A, B, A, C, A send us to the Rondo form, the ending in C, and the fact that there are two movements, made us believe that in fact this is an original form, conceived to achieve diversity (namely the two movements) as well as (tonal) unity of this construction; at the same time, counting the bars until Allegro, we came up with a total of 78, of which A and the bridge to B have 30 and the rest 48, 78 : 48 = 1.625, a number very close to the value of Φ. Then we counted the bars in Allegro, including those inserted in Andante, and we came up with 107, of which C and A have 41 and the rest until the end 166, 107 : 66 = 1.621, even closer to the value of Φ. These findings led us to the idea that there may be two Fibonacci strings that lie at the foundations of this architecture, the former being 6, 6, 12, 18, **30**, **48**, **78**, and the latter 2, 7, 9, 16, 25, **41**, **66**, **107**. The former, dividable by 6, is the sixfold multiplication of the basic Fibonacci string 1, 1, 2, 3, 5, 8, 13. The latter, with 8 terms, has a ratio between the last and the last but one (the whole and the larger part) closer to Φ. According to the features of the string, the ratios of the lesser terms become gradually more distanced from the value of Φ. Hence, in the former 48 : 30 = 1.6, and in the latter 66 : 41 = 1.61. So there could be an innovation in terms of form, where there occur two closely-related forms, meaning that the Andante is a three-part lied written $\frac{A}{a}\frac{B}{bb'}\frac{A}{a_v}$, plus a "bridge" of 20 bars, and the Allegro would be a sonata where C is the exposition with two ideas, one in Sib and another in sib, and in the development there occurs a divertissement consisting of the first idea in Andante. In the reprise both ideas in C occur in the same tonality, Sib. This resourcefulness is no exception with Mozart, as most of his works testify of the composer's constant intention to avoid patterns. We were saying lied and sonata, but, at the same time, the repetition of a refrain after each part (here, A) means that the rondo is the one which gives unity to the form. And, as a refinement, to the idea of unity which perhaps Mozart might have thought of, we can count the 10 bars of Andante, inserted in Allegro, with a double duration (as given by the indication of the agogic, ♩= 63 şi ♩= 120), coming up with a total of 117 bars instead of 107, and the Fibonacci string would be 9, 9, 18, 27, **45**, **72**, 117, which is actually the multiplication by 9 of the basic Fibonacci string, the golden section thus being in bar 45, on the highest note in the aria. This would be the ultimate proof of unity for the entire aria, based on the same Fibonacci string, in different multiplications (6 and 9 times). And there would also be another small sign which Mozart perhaps leaves to this purpose, namely adding a bar at the end, when to the final cadence group in Si ≅ with 4 bars are added 5, not 4, thus getting a total of 9 (8+1) and not 8, as it would have been natural according to a axial symmetry. But with 9 over all the equation $a_n = a_{n-1} + a_{n-2}$, is completed, that is, 117=72+45, hence entailing the symmetry of the golden section (117 : 72 = 1,625 $\approx \Phi$). Thus, we are entitled to assume that Mozart knowingly avoided a *static symmetry* in favour of a *dynamic symmetry*[4], as if he had meant to say:

[4] Matila Ghyka, ESTETICĂ ŞI TEORIA ARTEI (AESTHETICS AND ART THEORY), (Bucharest: Ed. Ştiinţifică şi Enciclopedică, 1981), p. 342.

here I multiplied the Fibonacci string by 9, so that the Andante sostenuto with 78 bars represent $\frac{2}{5}$ of the total of 195 bars, and the Allegro assai with 117 bars $\frac{3}{5}$ of the whole. The terms of these ratios also belong, as can be seen, to the Fibonacci string. Nevertheless, as exciting as our "interpretation" may be, it is a matter of possibility, of chance. The readers are therefore invited to make their own choice.

From the standpoint of the tonal level, one may observe on the outline of the form and the graphic representation of the group the tonal shifts towards the neutral element (0 – in the group table it occurs diagonally), which here is the tonality Si ≅ and with respect to which the other 10 are organised symmetrically.

If we have in view the fact that the group of congruence classes (mod 12) is isomorphic, beside the multitude of tonalities, and with the multitude of intervals, then the composition of the tonalities of this aria, as agreed upon, would be equal to $60 \equiv 0 \pmod{12}$.



Sib Fa Reb Fa Sib sib sol Mib sol Sib sib Sib

1 + 8 + 4 + 11 + 9 + 3 + 11 + 1 + 0 + 9 + 3

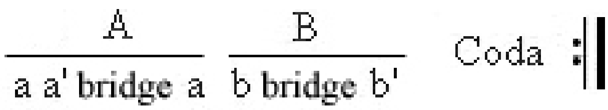
This means that the circle of fifths was run 5 times (60:12=5). It is a quantitative assessment, but it is worth considering since it expresses rigorously the great harmonic richness of this aria.

In Cunégonde's aria from the opera *Candide*, Leonard Bernstein (1918 – 1990) offers us the opportunity of a new encounter with the Fibonacci string.

In volume II of the cycle *Who's Afraid of Music History?* Ms Carmen Chelaru, a Romanian musicologist and doctor of music, regarded Bernstein's choice of Part I of the *Jupiter Symphony*, within the televised Concerts for youth, as highly significant in explaining the classical sonata form.

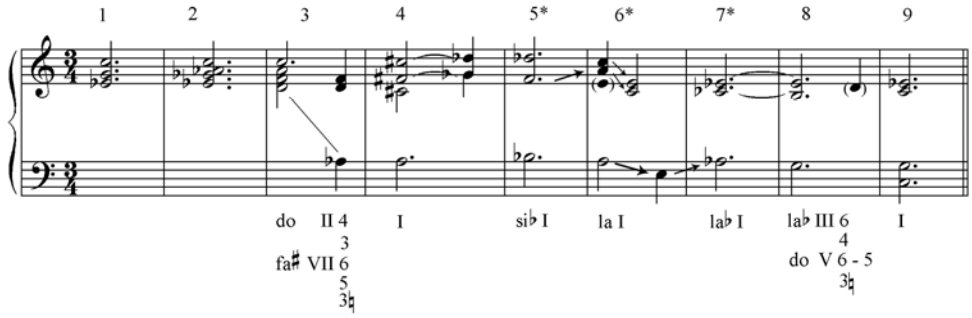
By analysing the aria from the American composer's opera, we shall see to what extent Mozart's oeuvre was influential as a model of minute elaboration of each detail, although the outer aspect of Bernstein's music makes us think rather of jazz music, a music of spontaneity and not by far one of mathematical rigorousness.

Its form is classical, with two repeated compound parts and a coda, which is enlarged the second time and is is obviously the fruit of an intense elaboration.



Evidently, this repeated form is required by the text which reflects the two contrasting moods of the character.

Therefore, we are not dealing with a three-part form, like in Mozart's aria, where the text had been a mere pretext. The dramatics demands here a certain mood at the end, and the recurrence of the initial atmosphere and mood are thus inappropriate. However, the inner structures of the form contain some very interesting harmonic or rhythmical elaborations.



The harmonic scheme above, of the first phrase of this aria, displays a skilful mastery of chromaticism and enharmonic modulation on 6 and 4 ascending fifths, respectively. Moreover, the three consecutive minor chords in bars 5, 6, 7 are simply juxtaposed within a descending semitone from one another (that is, in a chord sequence of 5 ascending fifths – sib la, lab), the parallelism of the voices being eluded by their ingenious conducting.

By modelling mathematically these modulations, and by imagining the table of the group (modulo 12) like we did with Mozart's aria, the neutral element (0) is in this case do, and the composition of the set elements would look as follows:

do fa# sib la lab do

$$0 + 6 + 4 + 5 + 5 + 4 = 24 \equiv 0 \pmod{12}$$

In order to render the heroine's sorrow, we can notice that the composer employed deliberately only minor tonalities – 5. Three of them belong to a diminished triad (do, fa#, la). This was short of a minor third, mib, to go back to the basic tonality. But less is more. The composer avoids the fourth minor third as well. When it comes to major thirds, their repeated succession shifts from **b** to **b'**; starting from Do, which was brought, through a tierce de Picardie, after do.

The composition of the set elements (mod 12) would look as follows:

Do Mi Lab

$$0 + 4 + 4 = 8 \pmod{12}$$

Lab is the tonality of **b'**. There should be another major third to go back to Do. The composer avoids this, since the aim is to go back to **A**, that is, to do. In this case, the composer modulates in other ways.

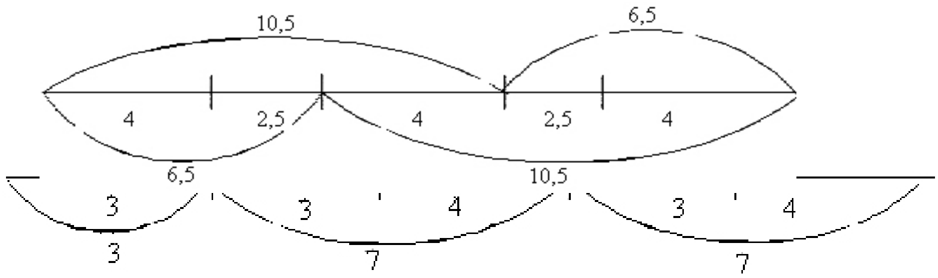
To conclude, **a** used minor thirds, and **b** used major thirds. Moreover, the tonalities are also major, and the intervals are large. Naturally, since the heroine's state of mind is totally different.

B's comeback is followed by an amplified coda (Un poco piu mosso) which brings about a relatively new period made up of the identical triple repetition of three bars in four beats and a final bar in five beats. That sums up to 17 beats. The bass, on a dominant pedal, marks the quavers by threes. It was all this that led us to the idea that the author wanted, eventually, the mirror symmetry with motifs and phrases as large as the powers of 2 to follow another order, dynamic, based on the Fibonacci string. Starting from a quaver (0.5 of a beat) we have constructed the Fibonacci string:

0,5; 0,5; 1; 1,5; 2,5; 4; **6,5**; **10,5**; 17.

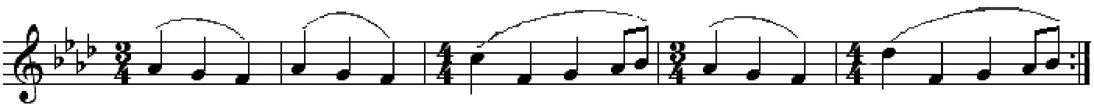
Here is where the 17 beats come from. 17:10,5 = 1,619 – one thousandth of Φ. We came quickly to this result. It is only natural. If we look closer, this string results from the division by 2 of the basic string: 1, 1, 2, 3, 5, 8, 13, 21, 34.

Here is the graph of this division conforming to the golden section:



The melodic cells (of crotchets) approximate this scheme.

If the composer had wished to highlight even better this division, the phrase could have been written as follows:



However, the composer wants another one, suggested by the legato and the metre: three bars in 4 beats and a final one in 5, as occurs in the score, that is, 4 * 3 + 5 = 17. Since 17 is repeated 3 times, the period has 51 beats. Thus can be explained the occurrence of the three-beat bar (126) added at the end of **b'**, apparently with no reason, to sum up to 51, that is, 48 + 3 = 51. In this way, the two sections have symmetric, "mirroring" durations, but on the inside, each has its own individual organisation.

We also have to underline that **b'** mostly contains a canon at the octave one beat apart. This and the polyrhythms in the coda, underlined intelligently by means of the orchestration, bring vivacity and exquisite delight to the ending of this aria.

The knowledge outside the musical area that has been used in this article to define the internal dialectics of the musical forms, structures and systems, of the harmony, counterpoint, theory, orchestration and aesthetics issues, they all must be put to serve the artistic achievement, and the pianist, fully mastering his or her technique, must fold entirely on the soloist's intentions, sometimes amplifying the limited strength of the voice by the sonorous force and amplitude of the piano, and some other times fading out discretely so that the audience may hear even the softest pianissimo. This is work which demands perfect collaboration. It is what we have tried to achieve.

But to what extent knowing these things bore any influence at all on the artistic quality of the recital we gave, this can only be stated by the audience present at the event, and this text (an excerpt from the recital presentation) has merely attempted to reveal one of the possible “interpretations” we assumed as “mediators” between the composer and the audience.

If the hypotheses proposed in this article may seem to some far-fetched, we shall resort, as a conclusion, to a quotation from Andrei Pleşu's *Călătorie în lumea formelor* (*A Voyage to the World of Forms*) from which one may understand that the very *spirit* occurring in Mozart's works, and which we found 200 years later in Bernstein's, actually has a wider range, “haunting” both the macrocosm as well as the microcosm, both the nature as well as the spirit of the art in general: “And if today the canvas is mastered with a casualness which is less observant of the rigorous calculations, let us not forget that the Villon brothers, Mondrian and, before them, Sérusier, they all knew what the gold section is and what it is used for.

And if, at one point, we shall discover that, let's say, Munch, sometimes creates according to the same canons as Botticelli, this does not necessarily mean that Munch read Alberti, but it might mean that his instinct followed the footsteps of a law well-recorded in his visual memory. Hence, our discovery stands. No artist is a stranger from the *spirit* that the just research of its *letter* stirs up, even if its letter is not explicitly or exclusively related to that spirit”[5].

Fibonacci perennis - From W.A. Mozart to L. Bernstein -

Junior lecturer **Andrei Hrubaru-Roată** PhD student
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Abstract

It is widely accepted that an Ariadne`s thread ties the antique Egypt’s monuments to the antique Greek’s temples and sculptures, the Renaissance’s picture and gothic cathedrals, the Picasso’s paintings and Le Corbusier`s architecture, and that this thread is in fact a proportion, the golden proportion. If this has relevance in the temporal arts, in music especially, it is an issue harder to cut. The intuition of space and time, although interconnected, presents undeniable features. In this studies we answer favorably to this dilemma, not from the esthetics point of view, but of the interpret, witch, having the privilege to be the one who lifts the first of the many veils that hide the mystery of creation embedded in the score, “playing” that mystery in the face of the audience, has the obligation to understand, as much as possible, and at a rational level, what where the subtle ins and outs of it , in what way they have the power to produce on the listener the desired effect imagined by the creator.

Counting on the known historic value of the two pieces of work in discussion here belonging to the representatives of two different eras, Mozart and Bernstein, and on the correctness of our judgment, we hope to have contributed, even in a small measure to the consolidation of our point of view that we have previously presented.

[5] Andrei Pleşu, CĂLĂTORIE ÎN LUMEA FORMELOR (A VOYAGE TO THE WORLD OF FORMS) (Bucharest: Meridiane, 1974), pp. 82-83.